The variation of accuracy with range

by David Lane

In my article in the April/May 1979 issue, 1 referred to the variation of accuracy with range as being fundamental to the construction of Handicap Tables. This article gives details of the results of Tournament Analysis carried out to find out what the variation is, in terms of practical shooting.

Before talking about the detailed results, some explanation of the general methods of analysis may be helpful.

If it is assumed that the distribution of shots on the target follows the circular normal distribution, then the average miss distance from the pin hole is a measure of the archer's accuracy and is directly related to the score which is obtained. Thus, as an archer improves, the average miss distance from the pin hole gets smaller and the score goes higher.

This can be looked at the other way round. If we know the score achieved on a given target we can estimate the average miss distance from the pin hole. From the average miss and the target range, we can calculate the average equivalent angular error from the archer's position on the shooting line.

This is illustrated in the following diagram:—

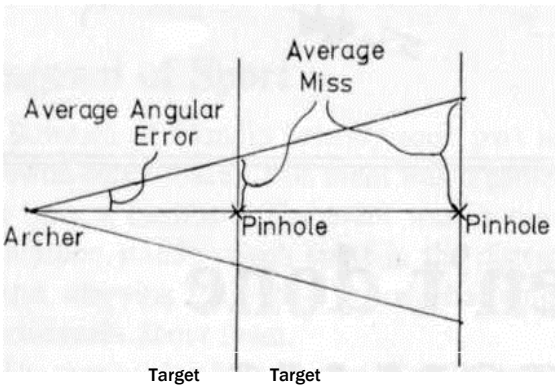

Fig 1.Average Miss & Average Angular Error

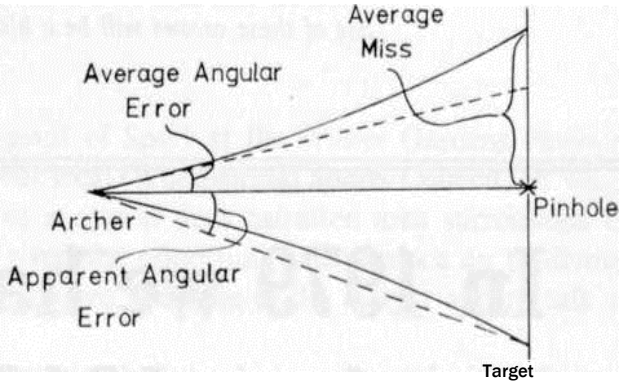

Fig.2 . Effect of Range

This average angular error may be taken as characteristic of the archer's ability and accuracy, and if arrows travelled in dead straight lines would give us a way of estimating what would happen when the target range is changed. Thus, if we doubled the target range, we would expect the miss distance from the pin hole to double and the score (on the same face) to be correspondingly reduced.

Unfortunately, life isn't quite as simple as this. Because of air retardation, the arrow slows down after it leaves the bow. Also, the elevation of the shot has to be changed for different ranges and hence the amount of time the arrow spends in the air is not directly proportional to target range. These effects result in the average miss increasing more rapidly as target range is increased than you would expect from Fig. 1. This is illustrated in Fig. 2.

The problem which has to be solved for the construction of handicap tables is to find out how much the average angular error increases with increased target range and whether the same relationship applies to both sexes, all age groups and all standards of archers.

The way in which this is done is by analysis of tournament results.

If we consider McKinney's performance in the 1977 World Championship, we find that his first round scores were:—

Total	90m	70m	50m	30m
1,231	276	287	328	340

These can be converted to average angular errors as follows:—

1.97 2.26 1.18 1.50 (milli-radians)

(Note 1 milli-radian is equal to 0.057 degrees or 3.44 minutes of arc).

From this it is clear that McKinney's angular accuracy in this particular tournament was worse at 70m than at 90m. There was a very significant improvement at 50m, but at 30m he was worse than at 50m.

If we now consider a rather lower level of performance — that of a boy shooting the men's FITA (Schoos in the 1978 Coupe d'Europe des Jeunes) we have:—

	Total	90m	70m	50m	30m
Score	706	124	163	168	251
Angular Error		5.26	5.45	4.87	4.81 (m. rad.)

Thus, in terms of average angular error, Schoos was shooting about 2½ to 3 times worse than McKinney and of course got a total score of 706 compared with McKinney's 1,231.

How can we reconcile these diverse results in terms of a general relationship which could be applied to both of them?

This can be done by dividing the angular error at 90m, 70m and 50m by the angular error at 30m and we obtain:—

	90m/30m	70m/30m	50m/30m	30m/30m
McKinney	1.31	1.51	0.79	1.00
Schoos	1.09	1.13	1.01	1.00

This way of expressing the results simply says that McKinney's average angular error at 90m was 32% worse than at 30m. At 70m, he was worse still - 51 %, but then had a good 50m which was 21 % better than his angular error at 30m.

Schoos - although shooting to a much lower standard overall was rather more consistent with errors of 9%, 13% and 1% worse than his 30m accuracy.

Similar examples can be given for Ladies and Girls, but these two are sufficient to illustrate the method and also to point out the types of variation which occur.

Firstly, all archers know the variation which can occur from one range to another — "90m was O.K., 70m was terrible, but it all came together again at 50m and it went like a dream". It is perhaps comforting to note that those remarks could be applied to McKinney's shooting in the results quoted above.

Secondly, it is clear that the weather and general environment of the tournament may have significant effects. Again, everyone (at least in this country) has experienced the wind getting up in the middle of the afternoon or the heavens opening!

Individual variations between one archer and another are ironed out by averaging over all of the archers in each tournament. Differences in conditions between one tournament and another are then ironed out by averaging over as many tournaments as possible.

Initially, all the different classes, ladies, gentlemen, girls, boys (in their age groups) are kept separate so that comparisons can be made.

The detail of these comparisons is too lengthy to report in this article, but in the end I decided that it was permissible to lump them all together. The final result is shown in Fig. 3.

Each of the crosses in Fig. 3. is the average over many archers and as many tournaments and classes as possible. The figure immediately below each cross gives the total number of tournaments

which have been averaged to produce the result and the complete diagram includes the analysis of over 1,400 individual archer's tournament results.

It will be seen that above 40m range, the results follow a reasonably smooth curve. These results are a mixture of adults and juniors. At 40m and ranges below 30m the results are, of course, for juniors only. There are very few junior results available and these show a wide scatter.

The English and Metric results blend together very nicely as perhaps we should expect, again with the exception of the short range results.

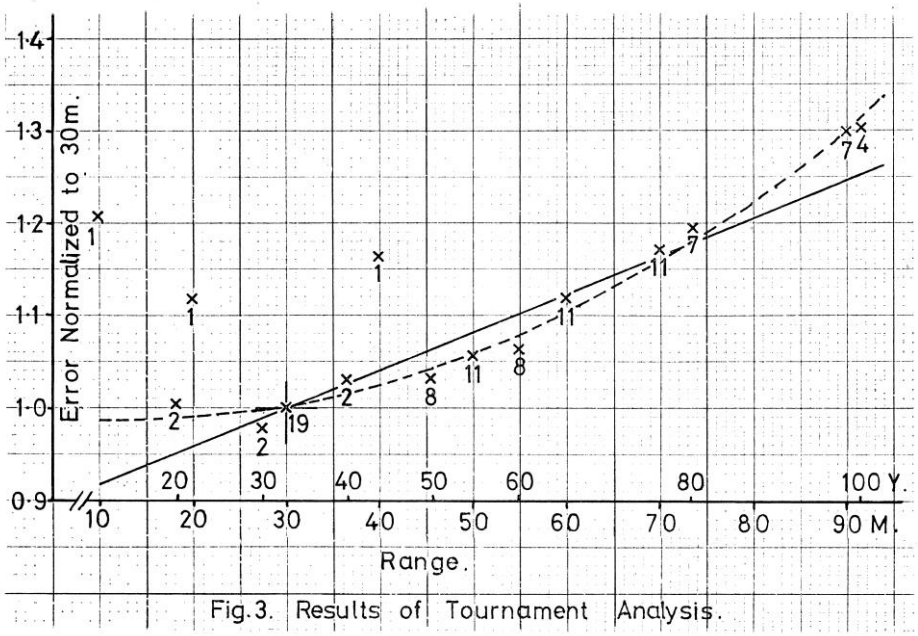

Fig. 3. Results of Tournament Analysis.

These short range results illustrate the fatigue effects to which I referred in my previous article. Remembering that all of the results are normalised to 30m, it is clear that some juniors are shooting up to 20% worse at ranges of 20m and 10m than they are at 30m. Perhaps part of the trouble is also that they are upset by the change to the 80cm face whereas the older juniors and adults have learnt that it doesn't make any difference. It may be significant that the results for 20 and 30 yards fit the general trend very much better than the results for 10 and 20m.

The diagonal straight line in Fig. 3 is the relationship which was built in to the 1973 Handicap Tables and a very similar relationship was used in my own Comprehensive Tables which have been circulated for comment.

However, from the results which have now been obtained, it appears that this relationship underestimates the errors at very short range and also at long range (90m and 100y.). Conversely, it overestimates the errors at the intermediate ranges from 50 to 60 yards including 50m. It was this latter result for 50m, before I had added in the English rounds, which led me to remark that there is a slight improvement in accuracy when shooting on the smaller 80cm. face. Now because of the way in which the English and Metric results blend together, I am changing my mind. The dotted curve also shown in Fig. 3 is a square law curve which has been fitted to the results. For the mathematically minded, this has been obtained by a least squares fit to all of the results including the junior results, weighted according to the number of tournaments in each average result.

Such a curve suggests that average angular error remains nearly constant to start with and then deteriorates at an ever increasing rate until it is about 30% worse at 90m than it is at 30m. This agrees very much better with the ballistics of a missile retarded by air resistance, than do the straight line relationships which have been used up until now.

The effect of using the dotted curve would be to modify the equivalent scores per dozen as illustrated in the following examples:—

	Score for 3 Doz			
	122 cm		80 cm	
	40m	30m	20m	10m
1973 Tables	144	201	207	297
Comprehensive Tables	143	201	203	291
New Curve	147	201	200	288

	Score for 3 Doz				
	122 cm			80 cm	
	90m	70m	60m	50m	30m
1973 Tables	184	235	260	234	300
Comprehensive Tables	181	233	260	234	300
New Curve	168	234	262	239	300

It will be seen that the new curve certainly eases the requirements on the juniors at very short range, and also eases the long range. Conversely, it increases the scores required at the intermediate ranges.

It would be interesting to know which of these results archers consider to be most realistic, but in view of the improved fit to actual tournament results, I would propose using the new type of curve for future issues of Handicap Tables.

However, I still have more tournaments to analyse and the results will continue to be accumulated so as to improve even this type of curve if possible. What is desperately needed of course is more results of junior tournaments in which the individual range results are kept separate in the Results Sheets.

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First Published in TOXOPHILUS Vol. II Number 3 Aug/Sep 1979

Reprinted Nov 2013